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FLOW SHOP SCHEDULING UNDER FUZZY ENVIRONMENT WITH RANDOM PROCESSING TIMES FOR MINIMUM SUM OF WAITING TIME OF JOBS

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Abstract

In earlier times Flow shop scheduling (FSS) in fuzzy background has acknowledged slight consideration. The paper presents the influence of the waiting time of jobs in a 2 machine k- job FSS in fuzzy environment. The times to process the jobs satisfies triangular fuzzy membership function. The main intention of the study is to find a sequence of jobs which delivers a least sum of the time of waiting of jobs. Heuristic approach has been adopted to achieve the desired objective. The experiments conducted for more than 2000 problems of various size for the problems with special structures and problems with random times of processing. The Weighted Mean Absolute error (WMAE) for the average of the sum of the times of waiting is computed for each job size which demonstrates that the presented step by step procedure of the Heuristic delivers significantly close to optimal solutions.

Keywords: Fuzzy, FSS, Heuristic, Processing Time, Special Structures, Waiting time.

1. INTRODUCTION

The Flow shop Scheduling (FSS) models examines the machine (service provider) - job (client) models for several objectives where all the jobs have to move in a pre-defined order of the machines. Several studies are available in the past to optimize the total completion time for n-job m-machine FSS problems but the objective to minimize the sum of the times of waiting of jobs has been paid less attention. The present paper explores the FSS models in fuzzy environment for 2-machine k-job problem where the objective is minimizing the sum of the times of waiting of jobs. The significance of the proposed objective can be detected in every single service provider organization/industry, where the client contentment is priority for every executive. In today's fastest rising world everyone desires to get the service deprived of waiting for too much time. Therefore, a service executive is constantly in attention to deliver service well-timed without making the client to wait for a long period. The wide-ranging literature emphases on deterministic times to process the jobs, so far there are many challenges in the real domain that comprises of uncertain consequences. Procedures that implemented on exact times of processing the jobs becomes fruitless to state the concerns that are centered on vagueness. In this paper a heuristic is anticipated to attain a sequence of jobs that will deliver an optimal or near optimal solution to the desired objective while considering the times to process the jobs under Fuzzy environment.

The techniques for defuzifying the fuzzy integers with triangular membership, has been delivered by Mc Cahon S. and Lee E.S. [1] using GMVs. Sanja P. and Xueyan S. [2] enhanced their outcomes by means of the strategy of α-cut with the aim to lessen the makespan in two machine FSS problems. In the present paper, Yager R.R. [3] ranking approach is employed to attain the preeminent results.

If the AHR value of times of processing the jobs satisfy a definite condition then FSS is considered as problem with special structures. The initial study for 2 machines k- job FSS problems with special structures was carried out by Bhatnagar V., Das G. and Mehta O.P. [4], the authors proposed an algorithm to optimize the sum of the times of waiting of jobs in FSS where the times of processing the jobs are not on the entire random but fulfilled a certain condition. The FSS problems with special structures considering the association of probabilities with the processing times has been deliberated by Gupta D. and Goyal B. [5] to optimize the waiting times of jobs. The study has been furthermore protracted by Gupta D. and Goyal B. [6] after making an allowance for the set-up times of machines detached from times of processing the jobs. Further Goyal B., Gupta D., Rani D., and Rani R. [7] made an addition to the study by making an allowance for the notion of job block and merited the recommended algorithm by making comparative analysis with the prevailing tactics.

Goyal B. and Kaur S. [8], [9] further explored the FSS models with times of processing in Fuzzy environment for the problems with special structures while considering the aim to optimize sum of the times of waiting of jobs.

The Heuristic tactic in exploring FSS models has been demonstrated a very operative tactic in research of Scheduling. Nawaz M., Enscore J. EE and Ham I. [10] introduced a well-known heuristic (NEH) algorithm with the intention to improve the total elapsed time for m-machine n- job FSS problem. Nailwal K.K., Gupta D. and Kawaljeet [11] developed two heuristics, one constructive and other an upgrading heuristic procedure in FSS for n- job, m –machine problem under no-wait constraint with the aim to lessen the makespan. Chakraborty U.K. and Laha D. [12] established a heuristic for FSS problem to attain an optimal schedule to minimize total ellapsed time/makespan. Recently the FSS problems in which time of processing the jobs was linearly dependent on waiting time of job has been studied by Yang, D. L and Kuo W.H. [13] to develop heuristic for the minimization of the makespan/ total elapsed time. Liang Z. PeisiZhong Liu M. Zhang C. and Zhang Z. [14] made a computational proficient optimization method though uniting NEH and NEH-NGA methods.

As per the Literature review it has been discovered that the aim of attaining the minimum of the sum of all the waiting time of jobs has been paid attention for the problems with special structures in fuzzy environment by Goyal B. and Kaur S. [8], [9]. The objective to minimize the sum of the waiting times of jobs for the problems with arbitrary times in fuzzy background has not been studied so far.

The present paper provides a heuristic to obtain an optimal or near optimal schedule with the aim to minimize sum of times of waiting of jobs. The problem with the aim to optimize the sum of the times of waiting of jobs is NP-Hard. So, a heuristic algorithm is proposed to optimize the total waiting time of jobs. In the present paper, a step by step procedure which can provide near optimal job schedule has been presented and the error analysis has also been deliberated.

2. PRELIMINARIES

2.1. Fuzzy Number: A fuzzy number $\tilde{N}$ is a convex fuzzy set of the real line R along with its membership function $\mu_{\tilde{N}}$: R→**[0,1]** which satisfies the following axioms:

(i) $\tilde{N}$ is normal, that is there exists exactly one $x \in R$ for which $\mu_{\tilde{N}}(x) = 1$.

(ii) $\mu_{\tilde{N}}(x)$ is piecewise continuous.

2.2. Triangular Fuzzy Number: A fuzzy number $\tilde{F} = (\beta_1, \beta_2, \beta_3)$ is said to be a triangular fuzzy number if it has membership function

$$\mu_{\tilde{F}}(x) = \begin{cases} \frac{x-\beta_1}{\beta_2-\beta_1}, & \beta_1 < x < \beta_2 \\ 1, & x = \beta_2 \\ \frac{\beta_3-x}{\beta_3-\beta_2}, & \beta_2 < x < \beta_3 \\ 0, & otherwise \end{cases} \tag{1}$$

Fig 1. Displays the Triangular membership function

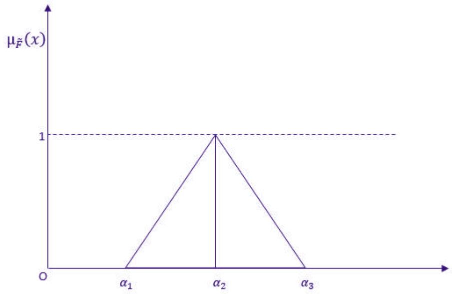


Fig. 1. Triangular Membership Fuzzy Number $\tilde{F}$=(β₁,β₂ ,β₃)

2.3. Yager R.R.[3] Ranking Method

For a triangular fuzzy number $\tilde{F}$, Yager R.R. [3] ranking index is given by

$$R(\tilde{F}) = \frac{1}{2}\int_0^1 (F_\beta^l + F_\beta^u)\, d\beta \tag{2}$$

Where (F_β^l, F_β^u) is the β-level cut for the fuzzy number $\tilde{F}$, $R(\tilde{F})$ is the Yager R.R. [3] ranking index for fuzzy number $\tilde{F}$.

2.4 Nomenclature

k : Number of jobs

p_j^i: The time of processing the j^{th} job on i^{th} machine in fuzzy background.

A_i : i^{th} Machine, $i = 1,2$

t_{ij} : AHR value of Time of processing j^{th} job on machine A_i $i = 1,2$

T_{ji} : Time of initial processing of j^{th} job on machine A_i

F_{ji} : Time of finishing the j^{th} job on machine A_i

W_j : Time of waiting of the j^{th} job on machine A_2

W : Sum of the times of waiting of k jobs on machine A_2

W_{best} : Most Accurate achieved value of W

W_{heu} : Achieved value of W obtained by executing recommended process

W_{max} : Maximum of all the possible values of W

3. MATHEMATICAL FORMULATION OF THE PROBLEM

3.1 FSS problem (2- machine k-job) having arbitrary times of processing:

Suppose that k-jobs are under process on two machines A_1 and A_2. All the jobs must be processed in the order A_1A_2. Let $p^1{}_j$, $p^2{}_j$ are the fuzzy times of processing the j^{th} job on machines A_1 and A_2 respectively. The matrix form of the problem has been given in Table 1.

Table 1: Problem description in matrix form

Job	Machine A_1	Machine A_2
j	p_j^1	p_j^2
1	$(\beta_{11}^1, \beta_{21}^1, \beta_{31}^1)$	$(\beta_{11}^2, \beta_{21}^2, \beta_{31}^2)$
2	$(\beta_{12}^1, \beta_{22}^1, \beta_{32}^1)$	$(\beta_{12}^2, \beta_{22}^2, \beta_{32}^2)$
3	$(\beta_{13}^1, \beta_{23}^1, \beta_{33}^1)$	$(\beta_{13}^2, \beta_{23}^2, \beta_{33}^2)$
.	.	.
.	.	.
.	.	.
k	$(\beta_{1n}^1, \beta_{2n}^1, \beta_{3n}^1)$	$(\beta_{1n}^2, \beta_{2n}^2, \beta_{3n}^2)$

The Yager R.R. [3] Ranking index of times of processing is denoted by t_{ij}.Let W_{σ_m} be the time of waiting for job σ_m on machine A_2. The purpose is to attain a sequence of jobs that optimizes the sum of times of waiting of jobs W.

3.2 FSS Problem with Special Structures:

In above problem, if the Yager R.R.[3] ranking index of times of processing the k jobs on machines A_1 and A_2 satisfies the condition

$$\max\{t_{1j}\} \le \min\{t_{2j}\} \quad (3)$$

Then, the FSS problem is recognized as 2-machine n-job with special structures FSS problem.

3.3 Assumptions:

1. Passing of jobs is not to be done.
2. Each process once underway must perform till end.
3. Jobs are self-regulating.
4. Job is not to be processed by more than one machines at a time
5. The time to set up a machine is supposed to be incorporated in times of processing the job.

3.4 Theorems

Lemma 3.4.a

Suppose that k-jobs are under process on two machines A_1 and A_2. All the jobs must be processed in the order A_1A_2. Let t_{1j}, t_{2j} are the times of processing the j^{th} job on machines A_1 and A_2 respectively. Let F_{jY} is the time of finishing of j^{th} job on machine A_2, then for job sequence $S: \gamma_1, \gamma_2, \gamma_3, \dots, \gamma_k$ of jobs

$$F_{\gamma_n 2} = \max_{1 \le v \le n}\left(\sum_{i=1}^{v} t_{1\gamma_i} + \sum_{j=v}^{n} t_{2\gamma_j}\right), \text{ where } n \in \{1,2,\dots,k\} \quad (4)$$

Proof: Let us smear the induction principle on S(n)

Where $S(n) = F_{\gamma_n 2} = \max_{1 \le v \le n}\left(\sum_{i=1}^{v} t_{1\gamma_i} + \sum_{j=v}^{n} t_{2\gamma_j}\right)$

Since $F_{\gamma_1 2} = t_{1\gamma_1} + t_{2\gamma_1}$.
Consequently, S(1) holds true.
Take up that S(n) holds true for $n = m$
Now for $n = m + 1$

$$F_{\gamma_{m+1}2} = \max(F_{\gamma_{m+1}1}, F_{\gamma_m 2}) + t_{2\gamma_{m+1}}$$

Using Hypothesis of Induction,

$$F_{\gamma_{m+1}2} = \max\left\{t_{1\gamma_1} + t_{1\gamma_2} + \dots + t_{1\gamma_{m+1}}, \max_{1 \le v \le m}\left(\sum_{i=1}^{v} t_{1\gamma_i} + \sum_{j=v}^{m} t_{2\gamma_j}\right)\right\} + t_{2\gamma_{m+1}}$$

$$= \max\left\{\sum_{i=1}^{m+1} t_{1\gamma_i} + t_{2\gamma_{m+1}}, \max_{1 \le v \le m}\left(\sum_{i=1}^{v} t_{1\gamma_i} + \sum_{j=v}^{m+1} t_{2\gamma_j}\right)\right\}$$

$$= \max_{1 \le v \le (m+1)}\left(\sum_{i=1}^{v} t_{1\gamma_i} + \sum_{j=v}^{m+1} t_{2\gamma_j}\right)$$

Hence for $n = m + 1$, $S(m + 1)$ holds true.
$S(n)$ comes out to be true for $n = 1$, $n = m$ and n= $m + 1$ and m being arbitrary.

Hence for any k-job sequence $S: \gamma_1, \gamma_2, \gamma_3, \dots, \gamma_k$, time of finishing of the job γ_n is certain to be obtained from equation (4).

Lemma 3.4.b

Following from Lemma 3.4.a, the same assumptions and notations, For the sequence $S: \gamma_1, \gamma_2, \gamma_3, \dots, \gamma_k$, of jobs $W_{\gamma_1} = 0$ and, for $2 \le m \le k$,

$$W_{\gamma_n} = \max\left\{0, \sum_{j=1}^{n-1} t_{2\gamma_j} - \min_{1 \le v \le (n-1)}\left(\sum_{i=v+1}^{n} t_{1\gamma_i} + \sum_{j=1}^{v-1} t_{2\gamma_j}\right)\right\} \quad (5)$$

where W_{γ_n} is the time of waiting for job γ_n on machine A_2

Proof: Since for any k-job sequence $S: \gamma_1, \gamma_2, \gamma_3, \dots, \gamma_k$, Time of waiting $W_{\gamma_n} = T_{\gamma_n 2} - F_{\gamma_n 1}$ for $1 \le n \le$ k.
For n=1

$$W_{\gamma_1} = T_{\gamma_1 2} - F_{\gamma_1 1} = t_{1\gamma_1} - t_{1\gamma_1} = 0$$

and for $2 \le n \le k$

$$W_{\gamma_n} = T_{\gamma_n 2} - F_{\gamma_n 1} = \max(F_{\gamma_n 1}, F_{\gamma_{n-1}2}) - F_{\gamma_n 1}$$

$$= \max(0, F_{\gamma_{n-1}2} - F_{\gamma_n 1})$$

Using equation (4)

$$W_{\gamma_n} = \max\left(0, \max_{1 \le v \le (n-1)}\left(\sum_{i=1}^{v} t_{1\gamma_i} + \sum_{j=v}^{n-1} t_{2\gamma_j}\right) - \sum_{i=1}^{n} t_{1\gamma_i}\right)$$

$$= \max\left(0, \max_{1 \le v \le (n-1)}\left(\sum_{i=1}^{n} t_{1\gamma_i} - \sum_{i=v+1}^{n} t_{1\gamma_i} + \sum_{j=1}^{n-1} t_{2\gamma_j} - \sum_{j=1}^{v-1} t_{2\gamma_j}\right) - \sum_{i=1}^{n} t_{1\gamma_i}\right)$$

$$= \max\left(0, \max_{1 \le v \le (n-1)}\left(-\sum_{i=v+1}^{n} t_{1\gamma_i} + \sum_{j=1}^{n-1} t_{2\gamma_j} - \sum_{j=1}^{v-1} t_{2\gamma_j}\right)\right)$$

$$= \max\left(0, \sum_{j=1}^{n-1} t_{2\gamma_j} + \max_{1 \le v \le (n-1)}\left(-\sum_{i=v+1}^{n} t_{1\gamma_i} - \sum_{j=1}^{v-1} t_{2\gamma_j}\right)\right)$$

$$= \max\left(0, \sum_{j=1}^{n-1} t_{2\gamma_j} - \min_{1 \le v \le (n-1)}\left(\sum_{i=v+1}^{n} t_{1\gamma_i} + \sum_{j=1}^{v-1} t_{2\gamma_j}\right)\right)$$

Theorem 3.4.c

Suppose that k-jobs are under process on two machines A_1 and A_2. All the jobs must be processed in the order A_1A_2. Let t_{1j} and t_{2j} are the times of processing the n jobs on machines A_1 and A_2 correspondingly. For any sequence $S: \gamma_1, \gamma_2, \gamma_3, \dots, \gamma_k$, the sum of the times of waiting of jobs (W) is agreed by

$$W = \sum_{t=2}^{k}\left[\max\left(0, \sum_{j=1}^{n-1} t_{2\gamma_j} - \min_{1 \le v \le (n-1)}\left(\sum_{i=v+1}^{n} t_{1\gamma_i} + \sum_{j=1}^{v-1} t_{2\gamma_j}\right)\right)\right] \quad (6)$$

Proof: For sequence of k-jobs $\gamma_1, \gamma_2, \gamma_3, \dots, \gamma_k$ $\quad W = \sum_{t=1}^{k} W_{\gamma_t}$

From lemma 3.4.b

$$W = \sum_{t=2}^{k}\left[\max\left(0, \sum_{j=1}^{n-1} t_{2\gamma_j} - \min_{1 \le v \le (n-1)}\left(\sum_{i=v+1}^{n} t_{1\gamma_i} + \sum_{j=1}^{v-1} t_{2\gamma_j}\right)\right)\right]$$

3.5 Inference of results for problems with special structures

Suppose that k-jobs are under process on two machines A_1 and A_2. All the jobs must be processed in the order A_1A_2. Let t_{1j} and t_{2j} are the times of processing the k jobs on machines A_1 and A_2 correspondingly which satisfies the relation assumed in equation (3)

$\max\{t_{1j}\} \le \min\{t_{2j}\}$

For any sequence $S: \gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_k$, the time of finishing of the job $F_{\gamma_n 2}$,deducted from Lemma 3.4.a is agreed by

$$F_{\gamma_n 2} = t_{1\gamma_1} + t_{2\gamma_1} + t_{2\gamma_2} \ldots + t_{2\gamma_n} \quad (7)$$

k-job sequence $S: : \gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_k$, $W_{\gamma_1} = 0$, and for $2 \le n \le k$, W_{γ_n} deducted from lemma 3.4.b is agreed by

$$W_{\gamma_n} = t_{1\gamma_1} + \sum_{s=1}^{n-1} y_{\gamma_s} - t_{1\gamma_n} \quad \text{where } y_{\gamma_s} = t_{2\gamma_s} - t_{1\gamma_s} \text{ and } \gamma_s \in \{1,2,\ldots,k\} \quad (8)$$

For k-job sequence :: $\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_k$, the sum of the times of waiting W deducted from theorem 3.4.c is agreed by

$$W = kt_{1\gamma_1} + \sum_{s=1}^{k-1} (k-s) y_{\gamma_s} - \sum_{j=1}^{k} t_{1j} \text{ , where } y_{\gamma_s} = t_{2\gamma_s} - t_{1\gamma_s} \text{ and } \gamma_s \in \{1,2,\ldots,k\} \quad (9)$$

Theorem 3.6

For $n \in \mathbb{N}$ and $y_1, y_2, \ldots, y_n \in \mathbb{R}$ such that $y_1 \le y_2 \le \ldots \le y_n$, the value $ny_1 + (n-1)y_2 + (n-2)y_3 + \ldots + 2y_{n-1} + y_n$ is minimum.

Proof: Using induction principle on n

Pettily result holds true for n=1

Take up that the result approaches true for upto n terms

Now,

$ny_1 + (n-1)y_2 + (n-2)y_3 + \ldots + 2y_{n-1} + y_n$

$= (n-1)y_1 + (n-2)y_2 + (n-3)y_3 + \ldots + y_{n-1} + \sum_{i=1}^{n} y_i$

The term $\sum_{i=1}^{n} y_i$ is constant,

Consequently following assumption to the hypothesis $ny_1 + (n-1)y_2 + (n-2)y_3 + \ldots + 2y_{n-1} + y_n$ is minimum.

4. ALGORITHMS

4.1 Algorithm for problems with special structures

The method, which is proficient in making ideal solution for a 2-machine k-job with special structures FSS problem under fuzzy environment, has been deliberated in recent times by Goyal B. and Kaur S. [8], [9]. Following is the depiction of algorithm in various steps:

Step1. Figure out the value of Ranking Index for fuzzy time of processing $p_j^i = (\beta_1, \beta_2, \beta_3)$, $j = 1,2,3, \ldots, k$ by means of the Yager R.R. [3] ranking index.

Step 2. Authenticate the structural relationship $\max\{t_{1j}\} \le \min\{t_{2j}\}$

Step 3. Compute $y_j = t_{2j} - t_{1j}$ for every j , where $j \in \{1,2,\ldots,k\}$

Step 4. Organize the jobs in arising direction of values of y_j. Suppose $S_1: (\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_k)$ is the sequence obtained after the arrangement.

Step 5. If $t_{1\gamma_1} = \min\{t_{1j}\}$, at that juncture the sequence acquired in the 3rd step is the requisite sequence which delivers ideal solution, or else move on to 5th step.

Step 6. Find $(k-1)$ altered sequences represented as S_j , for $2 \le j \le k$, by injecting j^{th} job in the first sequence to the very first place and leaving the left over sequence unaffected.

Step 7. Calculate the sum of the times of waiting of jobs, W for each and every sequences $S_1, S_2, \ldots, S_k$ with the help of equation (9).

Theorem 3.6 justifies that the sequence with least W is the requisite sequence.

4.2 Proposed Step by Step Heuristic

Following is the step by step description of algorithm:

Step1. Figure out the value of Ranking Index for fuzzy time of processing $p_j^i = (\beta_1, \beta_2, \beta_3)$, $j = 1,2,3, \ldots, k$ by means of the Yager R.R.[3] ranking index.

Step 2. Assemble the jobs according to the increasing order of times of processing t_{2j} of machine A_2 Assume the sequence attained is $\{\gamma_n\}$; $n = 1,2, \ldots k$

Step 3. Consider the initial two jobs from the obtained sequence and assemble them in both feasible ways to compute the times of waiting of jobs by means of formula given in the equation (6) and pick the sequence which provides least time of waiting for the initially selected two jobs.

Step 4. For $n = 3$ to k, go to 4th step and further to 5th step.

Step 5. Put in the n^{th} job in the attained sequence of (n-1) jobs at the n probable places begin to insert from 1st spot then to the 2nd spot and repeating the procedure.

Step 6. Compute the sum of the times of waiting of the jobs by means of formula obtained in equation (6) for the sequences obtained in 4th step and pick the sequence which delivers least sum of the times of waiting.

Tie breaking condition: If a tie persists among more than one partial sequences for the least sum of the times of waiting choice of that sequence, where the injection of the nth job comes at the far position should be done.

5. RESULTS AND ANALYSIS OF THE EXPERIMENTS

In order to estimate in general, the effectiveness for the proposed novel heuristic in the paper, numerous arbitrary problems are generated to operate. In the various trial conducted, approximately 2100 problems ranging from k =4 to 200 (For apiece size 100 problems and 21 dissimilar sizes) are being produced and tested.

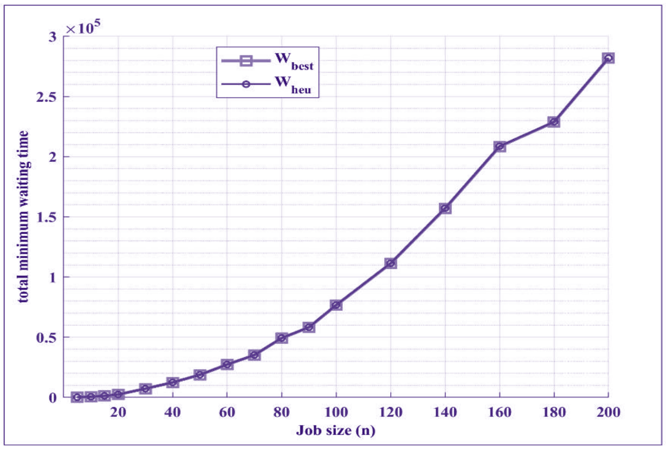
Table 2. Optimum sum of the waiting times for FSS problems with special structures

K	100 problems observed for each job size k	
	Average W_{best}	Average W_{heu}
5	85.32	85.95
10	474.12	476.06
15	1224.8	1229.3

20	2386.2	2395.4
30	7211.6	7224.8
40	12379	12409
50	18717	18770
60	27268	27313
70	35115	35194
80	49223	49304
90	58111	58250
100	76525	76703
120	111230	111400
140	156930	157200
160	208440	208680
180	228782	228962
200	281850	282010

The results obtained in Table 2 demonstrates that the sum of the Waiting times attained after applying the proposed heuristic provides the nearly optimal solution.

Fig. 2. Comparison of the Average of the Proposed results with the averages of Optimal solution



The graphical view of the Table 2 is presented in Fig. 2 which also demonstrates the efficiency of the proposed step by step procedure of the heuristic.

Table 3. Error Analysis for FSS problems with Special Structures

k	100 problems observed for each job size k
	WMAE
5	0.0073
10	0.0053
15	0.0035
20	0.0038
30	0.0019
40	0.0024
50	0.0027
60	0.0017
70	0.0021
80	0.0016
90	0.0023
100	0.0024
120	0.0014
140	0.0017
160	0.0013
180	0.0007
200	0.0005

Taking k number of jobs, Weighted Mean Absolute Error (WMAE) $e = \frac{\sum_{i=1}^{100} |W_{T(best)} - W_{T(heu)}|}{\sum_{i=1}^{60} W_{T(best)}}$ is computed in Table 3 where $W_{T(heu)}$ is the optimal of the sum of the waiting times obtained by employing the heuristic and $W_{T(best)}$ is the optimal of sum of waiting times of all jobs for all probable permutation of the schedules.

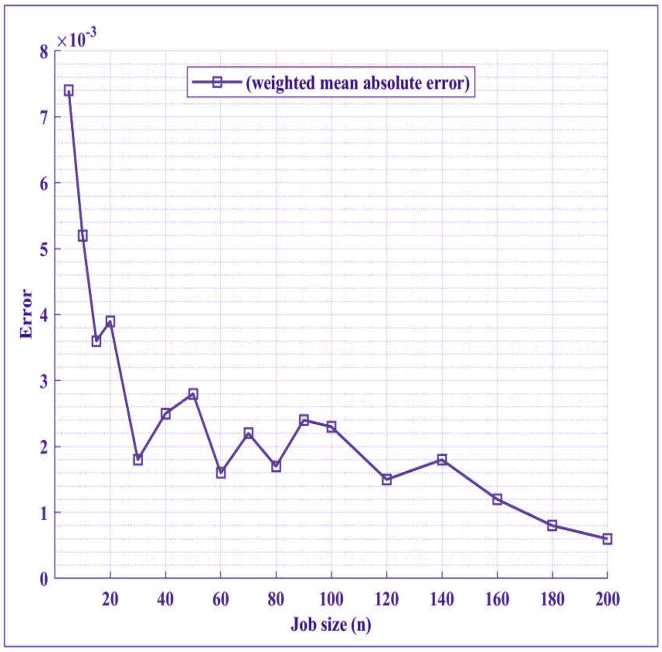


Fig. 3. WMAE for FSS problems with special structures

The results obtained in Table 3 are presented in Fig. 3 which demonstrates that the error reduces accordingly to the increase of the size of the job. Therefore the presented heuristic provides the near optimal solution significantly. Various trial conducted for the problems with arbitrary processing times, approximately 400 problems ranging from k=4 to 7 jobs are being produced and tested. The results obtained after applying the proposed heuristic are then compared with the actual least summation of the waiting times of jobs in Table 4. Also a maximum possible waiting time is computed to further prove the effectiveness of the presented heuristic. It is significantly proved that the **Average W_{heu}** is very much lower than **Average W_{max} (Fig. 4)**

Table 4. Average of the Sum of Time of waiting of jobs for FSS problems with arbitrary times of processing

k	Experimented conducted for 100 problems for each k		
	Average W_{best}	Average W_{heu}	Average W_{max}
4	11.97	13.01	91.17
5	26.54	28.79	196.54
6	31.37	34	333.07
7	33.32	36.11	275.91

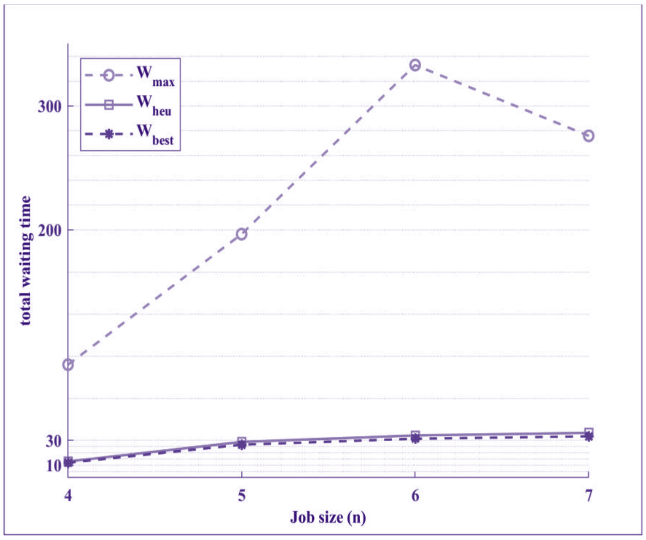


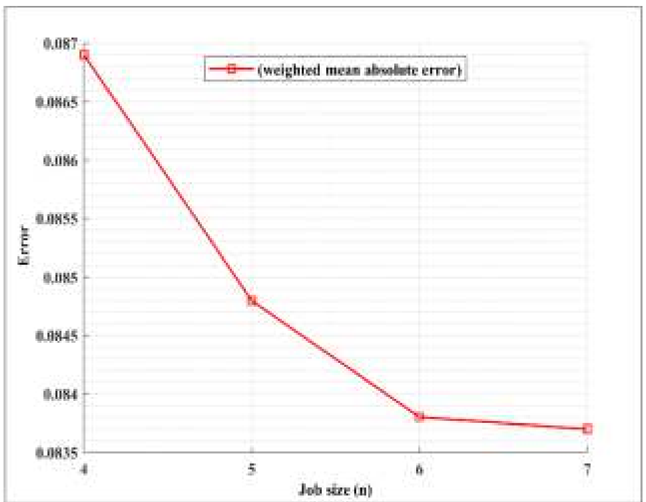
Fig. 4. Averages of Sum of the Times of waiting for FSS problems with arbitrary times of processing

The WMAE are also computed in Table 5 for the proposed heuristic which are proving that the presented step by step procedure is highly effective to reach near the optimal solution. Due to the NP-Hardness complexity of the problem actual results can only be generated up to job size k=7 but they are enough to demonstrate that the presented heuristic is providing lesser WMAE as the job size is increasing. Fig. 5 provides the graphical view of the data generated and presented in Table 5

Table 5. WMAE computation for problems with arbitrary times of processing

k	Experiment conducted for 100 problems for each k
	WMAE
4	0.0868
5	0.0847
6	0.0839
7	0.0838

Fig. 5. WMAE computation for problems with Arbitrary times of processing



6. CONCLUSION AND FURTHER SCOPE

In the paper a heuristic approach has been implemented with the aim to optimize the total of the waiting time of jobs. The algorithm developed using the heuristic approach is defensible by performing computational trials. The results found by the trials are also compared with the optimal results for several problems. The method for problems with special structures under fuzzy environment given by Goyal B. and Kaur S. [8], [9] provides the minimum of the waiting time. The proposed heuristic algorithm has been constructed basically to apply on to the problems with randomly generated processing times of both the machines. The computational trials and results show that the proposed algorithm delivers optimal or near optimal resolution to the problems with special structures as well. The proposed algorithm has been demonstrated to be operative not only for problems with special structures nevertheless for problems with arbitrary processing times also. The future work can be extended by detaching times of set up of machines from the times of processing the jobs.

7. DECLARATIONS

Conflicts of interests

The authors have no competing interests to declare that are relevant to the content of this article.

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9. CONTRIBUTION OF AUTHORS

Author Dr. Deepak Gupta formulated the problem and designed the algorithm. Author Bharat Goyal wrote the main manuscript text and conducted the computational experiments. Both the authors reviewed the manuscript.

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